Pre-Stage 1 Report
Aeroacoustic Sound Synthesis Models

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Abstract

Presented here is an informal document, reviewing and allowing self-evaluation of my work over the past three months. It is not intended for formal evaluation mearly as a reference and guide, where comments, thoughts and criticisms are welcome (and desired). The work includes current and future work and has been compiled as a pretext to the Stage 1 Report. It is not intentionally written in a formal style as it is believed more important to consolidate my thoughts and intended direction.

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1 Introduction

Over the past 3 months I have been looking at possible ways to synthesis the sound of wind as it interacts with objects in the environment. It was decided not to look at models that simply shape noise to create similar sounds but to rather look at the physics behind the sound making process. These mathematical principals would then be modelled in order to synthesis the correct sounds.

The start of my research took me to first read a general book on physics [1] as well as a number of general books on maths, [2], [3] and [4]. This was followed by a book on the topic of fluid dynamics [5] as well as an online module taught by Boston University [6].

This was all in order to refresh my previous maths knowledge in topics such as differential equations and linear algebra. The text on physics got me back into thinking about Newtons Laws as well as identifying systems. I am new to the topic of Fluid Dynamics and a general course helped me get familiar with the subject matter to a standard where I was able to start following the terminology and thinking behind the subtopic of aeroacoustics. The level of mathematics and physics that lie behind this subject is very high and it is fair to say the learning curve for this topic has been steep and, at times, challenging.

1.1 Aims and Objectives

The aim of my research is to create accurate synthesis models for aeroacoustic sounds. This is to be achieved by way of mathematically modelling the physical processes behind the sound creation. This is commonly known as Physical Modelling in the sound synthesis community.

It is a desired quality of the sound synthesis models that the are able to run in real time rather than have to rely on time consuming Computational Fluid Dynamics that are run offline. The models created should be as light weight as possible with respects to processing.

The justification of this research is the ability to create a physical model of aeroacoustic sounds that operates in real time will allow a number of sound effects to be created for the use in video games, film and television. Aeroacoustic sounds include wind sounds, swooshing sword sounds, jet engines, propellors to name a few.

The objectives of the research is as follows:

- Model wind sounds using the principals of fluid dynamics, including vortex sounds. This includes looking at the sounds of wind and poles, trees, buildings, doorways and other cavities (aeolian and cavity sounds)
- Model the sounds of an object being swung / thrown through the air. For example a golf club, sword, propellors or a book (aeolian sounds)
- Could it be used to provide a light weight mimic of musical instruments? (Both cavity and aeolian sounds)
- Can the models be re-formulated to other fluids, e.g. water?

The overlying principal behind all the models is to be able to create real-time procedurally generated sound effects for use in the entertainment industry. The benefit of these sounds are that they the rules behind the sound making process which can be easily manipulated or programmed to sound unique and accurate each time they are employed.

This is useful in the video game industry where the use of samples can be very repetitive and lack realism. Since the games are non-linear and the player can return to the same gameplay action repeatedly, unique and new sounds based on the physics of the environment offer a far more encapsulating experience.

For film and television the use of procedural sound effects is also becoming popular. It is hypothesised that this is due to the sound designers not having to physically go and record every required sound effect but can rather remain in their editing studio, adjusting the synthesis model parameters to their desired sound.

The sound of wind is an integral part of environmental sounds. As well as the interaction of the wind and the environment, the wind also has effect on other environmental components such as rain. It also affects the transmission of sounds like thunder. A realistic real-time physical model of the wind can be justified for this reason. Once the physics has been replicated, properties of the vortices created can be used as devices of feedback to the sound creation process. This happens naturally in some instances which explains whistling wind sounds. It is thought that knowledge of the vortices can also be utilised to control objects like flags and swinging cables that all contribute to environmental sounds.

The aeolian sounds can also be used as swoosh sounds as objects move rapidly through the air. This includes but is not exclusive to, sword sounds, clubs and bats, thrown objects, e.g. balls and books, propellors. The theory quite possibly breaks down here for a bullet which travels at speeds approximately just below 1 Mach to higher. It is believed different theory comes into play here and will include things like jets and explosions.

The sound sources to be modelled for the purposes stated above are for un-tuned, interactions with objects where the sound making properties are incidental to the design and purpose of them. But there are objects that the main design is to create sound in this way. This was found when researching aeolian sounds there is an instrument called the Aeolian Harp which wis a musical instrument played by the wind and named after the Greek god of wind, Aeolus. It is then concluded that this instrument could be modelled by the same models that are creating the wind interaction sounds. Extending this, can other wind driven instruments also be modelled, e.g. pipes, recorders, flutes, blowing over bottles, and other such acoustic wind instruments?

What adjustments would have to be made to these models to be useful for sounds in water rather than air? The speed of sound, viscosity and fluid density are all different. A large proportion of fluid sounds are a result of bubbles. The sound from the bubbles relies on Minnearts solution for the sound of bubbles. It might be possible that any vortex model could be part of a water model, as modelling some of the movements that cause the creation of bubbles as well as adding some sounds to the water effects.

2 Background and State of the Art

2.1 Fluid Dynamics

The following derivations are an expanded version from the one given in [7]. Some explanations are taken verbatum from the text.

2.1.1 Lighthills Analogy

The fundemental equations in are the equations of mass conservation, momentum conservation and energy conservation. The mass conservation equation is given in equation 1:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}(\rho \vartheta_i) = m \tag{1}$$

where ρ is the fluid density, ϑ is the flow velocity at position x and time t. The momentum conservation equation is given in equatio‘2:

$$\frac{\partial}{\partial t}(\rho \vartheta_i) + \frac{\partial}{\partial x_i}(P_{ij} - \rho \vartheta_i \vartheta_j) = f_i + m\vartheta_i \tag{2}$$

where f_i is an external force density, P_{ij} is minus the fluid stress tensor, ($P_{ij} = p\delta_{ij} - \tau_{ij}$, where p is pressure, δ_{ij} is the Kronecker delta and τ is the viscous stress tensor), and $m\vartheta_i$ is the momentum added by the issuing mass. To derive Lighthill's wave equation from equations 1 and 2 we take the following steps:

Take the time derivative of the mass conservation equation, (equation 1) to get equation 3:

$$\frac{\partial^2}{\partial t \partial x_i}(\rho\vartheta_i) = \frac{\partial m}{\partial t} - \frac{\partial^2 \rho}{\partial t^2} \tag{3}$$

We can write the mass m as a density ρ_m of volume fraction $\beta = \beta(\mathbf{x}, t)$ injected at a rate

$$m = \frac{\partial}{\partial t}(\beta\rho_m) \tag{4}$$

therefore equation 3 becomes

$$\frac{\partial^2}{\partial t \partial x_i}(\rho\vartheta_i) = -\frac{\partial^2 \rho_f}{\partial t^2} + \frac{\partial^2(\beta\rho_f)}{\partial t^2} \tag{5}$$

where ρ_f is the total fluid density, $\rho_f = \rho_0 + \rho'$, ρ_0 is the steady state fluid density and ρ' is the density fluctuations. If we now look at the momentum conservation equation 2, the first step we take is the divergence of the equaiton:

$$\frac{\partial^2}{\partial t \partial x_i}(\rho\vartheta_i) = -\frac{\partial^2}{\partial x_i \partial x_j}(P_{ij} - \rho\vartheta_i\vartheta_j) - \frac{\partial f_i}{\partial x_i} \tag{6}$$

Hence from equation 5 and equation 6 it can be seen that

$$\frac{\partial^2 \rho_f}{\partial t^2} = \frac{\partial^2}{\partial x_i \partial x_j}(P_{ij} - \rho\vartheta_i\vartheta_j) + \frac{\partial^2(\beta\rho_f)}{\partial t^2} + \frac{\partial f_i}{\partial x_i} \tag{7}$$

Since only ρ' varies with time, a wave equation for ρ' can be constructed. First, the term $c_0^2(\partial^2\rho'/\partial x_i^2)$ is subtracted from both sides of equation 6 to get equation 8. c_0 is the speed of sound at the observer.

$$\frac{\partial^2}{\partial t \partial x_i}(\rho\vartheta_i) - c_0^2(\frac{\partial^2 \rho'}{\partial x_i^2}) = -\frac{\partial^2}{\partial x_i \partial x_j}(P_{ij} - \rho\vartheta_i\vartheta_j) + \frac{\partial f_i}{\partial x_i} - c_0^2(\frac{\partial^2 \rho'}{\partial x_i^2}) \tag{8}$$

This then makes equation 7 now:

$$\frac{\partial^2 \rho_f}{\partial t^2} - c_0^2(\frac{\partial^2 \rho'}{\partial x_i^2}) = \frac{\partial^2}{\partial x_i \partial x_j}(P_{ij} - \rho\vartheta_i\vartheta_j) - c_0^2(\frac{\partial^2 \rho'}{\partial x_i^2}) + \frac{\partial^2(\beta\rho_m)}{\partial t^2} - \frac{\partial f_i}{\partial x_i} \tag{9}$$

We can make use of the relationship given in equation 10:

$$c_0^2(\frac{\partial^2 \rho'}{\partial x_i^2}) = \frac{\partial^2(c_0^2\rho'\delta_{ij})}{\partial x_i^2} \tag{10}$$

where δ_{ij} is the Kronecker delta. This allows us to define the equation of Lighthill, equation 11.

$$\frac{\partial^2 \rho_f}{\partial t^2} - c_0^2(\frac{\partial^2 \rho'}{\partial x_i^2}) = \frac{\partial^2 \mathbf{T}_{ij}}{\partial x_i \partial x_j} + \frac{\partial^2(\beta\rho_m)}{\partial t^2} - \frac{\partial f_i}{\partial x_i} \tag{11}$$

where $\mathbf{T}_{ij}$ is Lighthill's stress tensor, defined by

$$\mathbf{T}_{ij} = \mathbf{P}_{ij} + \rho\vartheta_i\vartheta_j - (c_o^2\rho' + p_0)\delta_{ij} \tag{12}$$

Using the relationship $P_{ij} = p\delta_{ij} - \tau_{ij}$, equation 12 can also be written as:

$$\mathbf{T}_{ij} = \rho\vartheta_i\vartheta_j - \tau_{ij} + (p' - c_o^2\rho')\delta_{ij} \tag{13}$$

Equation 13 enables three basic aero-acoustic processes which result in sound. The Reynolds Stress Tensor, $\rho\vartheta_i\vartheta_j$, describes non-linear convective forces. There are viscous sources, τ_{ij} and the deviation from a uniform sound velocity c_0 or the deviation from an isentropic behaviour, $(p' - c_o^2\rho')\delta_{ij}$

2.1.2 Green's Functions

Green's Theorem allows us to construct an integral equation which combines the effect of sources, propagation, boundary conditions and initial conditions in a simple formula. The Greens function $G(\mathbf{x}, t|\mathbf{y}, \tau)$ is the pulse response of the wave equation and shown in equation 14.

$$\frac{\partial^2 G}{\partial t^2} - c_0^2\frac{\partial^2 G}{\partial x_i^2} = \delta(\mathbf{x} - \mathbf{y})\delta(t - \tau) \tag{14}$$

where G is the Green's Function, The pulse $\delta(\mathbf{x} - \mathbf{y})\delta(t - \tau)$ is released at the source point $\mathbf{y}$ at time τ and G is measured at the observation point $\mathbf{x}$ at time t. The Green's Function also satisfies equation 15 due to its reciprocity relationship and the symmetry of the Kronecker delta.

$$\frac{\partial^2 G}{\partial \tau^2} - c_0^2\frac{\partial^2 G}{\partial y_i^2} = \delta(\mathbf{x} - \mathbf{y})\delta(t - \tau) \tag{15}$$

From this we shall solve the wave equation:

$$\frac{\partial^2 \rho'}{\partial \tau^2} - c_0^2\frac{\partial^2 \rho'}{\partial y_i^2} = q(\mathbf{y}, \tau) \tag{16}$$

where $q(\mathbf{y}, \tau)$ is the sound source. The first step is to multiple equation 15 by $\rho'(\mathbf{y}, \tau)$. This gives equation 17:

$$\rho'(\mathbf{y}, \tau)\frac{\partial^2 G}{\partial \tau^2} - \rho'(\mathbf{y}, \tau)c_0^2\frac{\partial^2 G}{\partial y_i^2} = \rho'(\mathbf{y}, \tau)\delta(\mathbf{x} - \mathbf{y})\delta(t - \tau) \tag{17}$$

Equation 16 is multiplied by $G(\mathbf{x}, t|\mathbf{y}, \tau)$ to give equation 18:

$$G(\mathbf{x}, t|\mathbf{y}, \tau)\frac{\partial^2 \rho'}{\partial \tau^2} - G(\mathbf{x}, t|\mathbf{y}, \tau)c_0^2\frac{\partial^2 \rho'}{\partial y_i^2} = q(\mathbf{y}, \tau)G(\mathbf{x}, t|\mathbf{y}, \tau) \tag{18}$$

Equation 19 is found by subtracting equation 17 from equation 18:

$$\begin{gathered} G\frac{\partial^2 \rho'}{\partial \tau^2} - \rho'(\mathbf{y}, \tau)\frac{\partial^2 G}{\partial \tau^2} - Gc_0^2\frac{\partial^2 \rho'}{\partial y_i^2} + \rho'(\mathbf{y}, \tau)c_0^2\frac{\partial^2 G}{\partial y_i^2} = \\ -\rho'(\mathbf{y}, \tau)\delta(\mathbf{x} - \mathbf{y})\delta(t - \tau) + q(\mathbf{y}, \tau)G(\mathbf{x}, t|\mathbf{y}, \tau) \end{gathered} \tag{19}$$

Re-arranging gives:

$$\begin{gathered} \rho'(\mathbf{y}, \tau)\delta(\mathbf{x} - \mathbf{y})\delta(t - \tau) = q(\mathbf{y}, \tau)G(\mathbf{x}, t|\mathbf{y}, \tau) + \\ \rho'(\mathbf{y}, \tau)\frac{\partial^2 G}{\partial \tau^2} - G\frac{\partial^2 \rho'}{\partial \tau^2} - c_0^2\left(\rho'(\mathbf{y}, \tau)\frac{\partial^2 G}{\partial y_i^2} - G\frac{\partial^2 \rho'}{\partial y_i^2}\right) \end{gathered} \tag{20}$$

Equation 20 is then integrated to $\mathbf{y}$ over volume, V and to τ between t_0 to $t+$.

$$
\begin{aligned}
\rho'(\mathbf{x},t) &= \int_{t_0}^{t+} \iiint_V q(\mathbf{y},\tau) G(\mathbf{x},t|\mathbf{y},\tau)\, d\mathbf{y}\, d\tau \\
&+ \int_{t_0}^{t+} \iiint_V \left[\rho'(\mathbf{y},\tau)\frac{\partial^2 G}{\partial \tau^2} - G\frac{\partial^2 \rho'(\mathbf{y},\tau)}{\partial \tau^2}\right] d\mathbf{y}\, d\tau \\
&- c_0^2 \int_{t_0}^{t+} \iiint_V \left[\rho'(\mathbf{y},\tau)\frac{\partial^2 G}{\partial y_i^2} - G\frac{\partial^2 \rho'(\mathbf{y},\tau)}{\partial y_i^2}\right] d\mathbf{y}\, d\tau
\end{aligned} \tag{21}
$$

Partial integration over the time of the second integral and over the space of the third integral in the right-hand side of (21) gives:

$$
\begin{aligned}
\rho'(\mathbf{x},t) &= \int_{t_0}^{t+} \iiint_V q(\mathbf{y},\tau) G(\mathbf{x},t|\mathbf{y},\tau)\, d\mathbf{y}\, d\tau \\
&- c_0^2 \int_{t_0}^{t+} \iint_S \left[\rho'(\mathbf{y},\tau)\frac{\partial^2 G}{\partial y_i^2} - G\frac{\partial^2 \rho'(\mathbf{y},\tau)}{\partial y_i^2}\right] n_i\, d\mathbf{y}\, d\tau \\
&- \left[\iiint_V \left[\rho'(\mathbf{y},\tau)\frac{\partial^2 G}{\partial \tau^2} - G\frac{\partial^2 \rho'}{\partial \tau^2}\right] d\mathbf{y}\right]_{\tau=t_0}
\end{aligned} \tag{22}
$$

where n_i is the outer normal to the surface S. The second integral vanishes for a tailored Greens function and the third integral represents the effect of the initial conditions at $\tau = t_0$. For a tailored Greens function, and if $t_0 = -\infty$, we have the superposition principle over elementary sources which we expect intuitively:

$$
\rho'(\mathbf{x},t) = \int_{t_0}^{t+} \iiint_V q(\mathbf{y},\tau) G(\mathbf{x},t|\mathbf{y},\tau)\, d\mathbf{y}\, d\tau \tag{23}
$$

2.1.3 Curle's Method

Curle's method is a method of calculating the sound radiation when a compact body is present in a flow. It makes use of Lighthill's Theory and the free field Green's function G_0 which implies that we should take surface contributions in equation 22 into account, (the second integral). An advantage of Curle's Method is that we can still use the symmetry properties of G_0 like:

$$
\frac{\partial G_0}{\partial y_i} = -\frac{\partial G_0}{\partial x_i} \tag{24}
$$

The surface terms have for compact rigid bodies quite simple physical meaning. The pulsation of the volume of the body is a volume source while the force on the body is an aero-acoustic dipole. In this way we can in fact say that if we know the aerodynamic (lift and drag) force on a small propeller we can represent the system by the reaction force acting on the fluid as an aero-acoustic source, ignoring further the presence of the body in the calculation of the radiation.

The first step in this method is to look at equations 11 and 16, we can see that

$$
q(\mathbf{y},\tau) = \frac{\partial^2 T_{ij}}{\partial y_i \partial y_j} + \frac{\partial^2 (\beta \rho_m)}{\partial t^2} - \frac{\partial f_i}{\partial y_i} \tag{25}
$$

Ignoring external mass sources and force fields gives:

$$
q(\mathbf{y},\tau) = \frac{\partial^2 T_{ij}}{\partial y_i \partial y_j} \tag{26}
$$

Substituting for $q(\mathbf{y},\tau)$ in equation 22 and taking $t_0 = -\infty$ gives:

$$\rho' = \int_{-\infty}^{t+} \iiint_V \frac{\partial^2 T_{ij}}{\partial y_i \partial y_j} G(\mathbf{x}, t|\mathbf{y}, \tau)\, d\mathbf{y}\, d\tau$$
$$-c_0^2 \int_{-\infty}^{t+} \iint_S \left[\rho'(\mathbf{y}, \tau) \frac{\partial^2 G}{\partial y_i^2} - G \frac{\partial^2 \rho'(\mathbf{y}, \tau)}{\partial y_i^2}\right] n_i\, d\mathbf{y}\, d\tau \tag{27}$$

Intergration by parts:

$$\int u \frac{dv}{dy}\, dy = uv - \int v \frac{du}{dy}\, dy \tag{28}$$

applied to the first integral in 33:

$$\int \frac{\partial T_{ij}}{\partial y_i \partial y_j} G_0\, d\mathbf{y} \tag{29}$$

where $u = G_0$ and $v = T_{ij}$.

$$\int \frac{\partial T_{ij}}{\partial y_i \partial y_j} G_0\, d\mathbf{y} = G_0 \frac{\partial T_{ij}}{\partial y_i} - \int \frac{\partial T_{ij}}{\partial y_j} \frac{\partial G_0}{\partial y_i}\, d\mathbf{y} \tag{30}$$

Partial integration is then applied to the integral in the right hand side of equation 30, where $u = \frac{\partial G_0}{\partial y_i}$ and $v = T_{ij}$.

$$\int \frac{\partial G_0}{\partial y_i} \frac{\partial T_{ij}}{\partial y_j}\, d\mathbf{y} = T_{ij} \frac{\partial G_0}{\partial y_j} - \int T_{ij} \frac{\partial^2 G_0}{\partial y_i \partial y_j}\, d\mathbf{y} \tag{31}$$

therefore the complete equation is:

$$\int \frac{\partial T_{ij}}{\partial y_i \partial y_j} G_0\, d\mathbf{y} = \int T_{ij} \frac{\partial^2 G_0}{\partial y_i \partial y_j}\, d\mathbf{y} + G_0 \frac{\partial T_{ij}}{\partial y_i} - T_{ij} \frac{\partial G_0}{\partial y_j} \tag{32}$$

Equation 33 then becomes:

$$\rho' = \int_{t-\infty}^{t+} \iiint_V T_{ij} \frac{\partial^2 G_0}{\partial y_i \partial y_j}\, d\mathbf{y}\, d\tau + \int_{-\infty}^{t+} \iint_S \left[G_0 \frac{\partial T_{ij}}{\partial y_i} n_i - T_{ij} \frac{\partial G_0}{\partial y_j} n_i\right]$$
$$-c_0^2 \left[\rho'(\mathbf{y}, \tau) \frac{\partial^2 G}{\partial y_i^2} n_i - G \frac{\partial^2 \rho'(\mathbf{y}, \tau)}{\partial y_i^2} n_i\right] d\sigma\, d\tau \tag{33}$$

Now we use equation 12 to substitute values for T_{ij}, where p_0 has no effect (?)

$$\rho' = \int_{-\infty}^{t+} \iiint_V T_{ij} \frac{\partial^2 G_0}{\partial y_i \partial y_j}\, d\mathbf{y}\, d\tau$$
$$+ \int_{-\infty}^{t+} \iint_S \left[G_0 \frac{\partial (P_{ij} + \rho \vartheta_i \vartheta_j - C_0^2 \rho'(\mathbf{y}, \tau))}{\partial y_i} n_i - (P_{ij} + \rho \vartheta_i \vartheta_j - C_0^2 \rho'(\mathbf{y}, \tau)) \frac{\partial G_0}{\partial y_j} n_i\right] \tag{34}$$
$$-c_0^2 \left[\rho'(\mathbf{y}, \tau) \frac{\partial^2 G}{\partial y_i^2} n_i - G \frac{\partial^2 \rho'(\mathbf{y}, \tau)}{\partial y_i^2} n_i\right] d\sigma\, d\tau$$

simplifying gives:

$$\rho' = \int_{-\infty}^{t+} \iiint_V T_{ij} \frac{\partial^2 G_0}{\partial y_i \partial y_j}\, d\mathbf{y}\, d\tau + \int_{t_0}^{t+} \iint_S \left[G_0 \frac{\partial (P_{ij} + \rho \vartheta_i \vartheta_j)}{\partial y_i} n_i\right] d\sigma\, d\tau$$
$$- \int_{-\infty}^{t+} \iint_S \left[(P_{ij} + \rho \vartheta_i \vartheta_j) \frac{\partial G_0}{\partial y_j} n_i\right] d\sigma\, d\tau \tag{35}$$

Using equation 2 and setting external forces to 0:

$$\frac{\partial}{\partial \tau}(\rho\vartheta_i) + \frac{\partial}{\partial y_i}(P_{ij} - \rho\vartheta_i\vartheta_j) = 0 \tag{36}$$

and the symmetry of G_0, (equation 24):

$$\begin{aligned}\rho' = \int_{-\infty}^{t+} \iiint_V T_{ij} \frac{\partial^2 G_0}{\partial x_i \partial x_j}\, d\mathbf{y}\, d\tau - \int_{-\infty}^{t+} \iint_S \left[G_0 \frac{\partial \rho\vartheta_i}{\partial \tau} n_i\right] d\sigma\, d\tau \\ + \int_{-\infty}^{t+} \iint_S \left[(P_{ij} + \rho\vartheta_i\vartheta_j)\frac{\partial G_0}{\partial x_j} n_i\right] d\sigma\, d\tau\end{aligned} \tag{37}$$

The spatial derivatives $(\partial/\partial x_i)$ do not refer to $\mathbf{y}$ and can be moved out of the integral. This gives:

$$\begin{aligned}\rho' = \frac{\partial}{\partial x_i \partial x_j} \int_{-\infty}^{t+} \iiint_V T_{ij} G_0\, d\mathbf{y}\, d\tau - \int_{-\infty}^{t+} \iint_S \left[G_0 \frac{\partial \rho\vartheta_i}{\partial \tau} n_i\right] d\sigma\, d\tau \\ + \frac{\partial}{\partial x_j} \int_{-\infty}^{t+} \iint_S [(P_{ij} + \rho\vartheta_i\vartheta_j) G_0 n_i]\, d\sigma\, d\tau\end{aligned} \tag{38}$$

In the far field the spatial derivatives $(\partial/\partial x_i)$ can be approximated by the time derivatives $-(x_j/|\mathbf{x}|)C_0^{-1}(\partial/\partial t)$. Equation 38 then becomes:

$$\begin{aligned}\rho' \simeq \frac{\partial x_i x_j}{|\mathbf{x}|^2 c_0^2} \frac{\partial^2}{\partial t^2} \int_{-\infty}^{t+} \iiint_V T_{ij} G_0\, d\mathbf{y}\, d\tau - \int_{-\infty}^{t+} \iint_S \left[G_0 \frac{\partial \rho\vartheta_i}{\partial \tau} n_i\right] d\sigma\, d\tau \\ - \frac{x_j}{c_0|\mathbf{x}|} \frac{\partial}{\partial t} \int_{-\infty}^{t+} \iint_S [(P_{ij} + \rho\vartheta_i\vartheta_j) G_0 n_i]\, d\sigma\, d\tau\end{aligned} \tag{39}$$

Employing integration by parts, (equation 28) on the second integral to move the time derivative $(\partial/\partial\tau)$ from $\rho\vartheta_i$ towards G_0, giving:

$$\begin{aligned}\rho' \simeq \frac{\partial x_i x_j}{|\mathbf{x}|^2 c_0^2} \frac{\partial^2}{\partial t^2} \int_{-\infty}^{t+} \iiint_V T_{ij} G_0\, d\mathbf{y}\, d\tau + \int_{-\infty}^{t+} \iint_S \left[\rho\vartheta_i \frac{\partial G_0}{\partial \tau} n_i\right] d\sigma\, d\tau \\ - \iint_S G_o \rho\vartheta_i\, d\sigma\, d\tau - \frac{x_j}{c_0|\mathbf{x}|} \frac{\partial}{\partial t} \int_{-\infty}^{t+} \iint_S [(P_{ij} + \rho\vartheta_i\vartheta_j) G_0 n_i]\, d\sigma\, d\tau\end{aligned} \tag{40}$$

The third integral in over the surface only and this disregarded as it is not time varying. (?) Green's Functions have a general symmetry in the time coordinate, (The effect of listening later is the same as shooting earlier)

$$\frac{\partial G}{\partial t} = -\frac{\partial G}{\partial \tau} \tag{41}$$

Using 41, equation 42 is obtained:

$$\begin{aligned}\rho' \simeq \frac{\partial x_i x_j}{|\mathbf{x}|^2 c_0^2} \frac{\partial^2}{\partial t^2} \int_{-\infty}^{t+} \iiint_V T_{ij} G_0\, d\mathbf{y}\, d\tau - \frac{\partial}{\partial t} \int_{-\infty}^{t+} \iint_S \rho\vartheta_i G_0 n_i\, d\sigma\, d\tau \\ - \frac{x_j}{c_0|\mathbf{x}|} \frac{\partial}{\partial t} \int_{-\infty}^{t+} \iint_S [(P_{ij} + \rho\vartheta_i\vartheta_j) G_0 n_i]\, d\sigma\, d\tau\end{aligned} \tag{42}$$

G_0 has the following property:

$$G_0 = \frac{\delta(t - \tau - r/c_o)}{4\pi r c_0^2} \tag{43}$$

which we use to carry out the time integrals and obtain Curle's Theorem:

$$\rho' \simeq \frac{\partial x_i x_j}{4\pi|\mathbf{x}|^2 c_0^4}\frac{\partial^2}{\partial t^2}\iiint_V \left[\frac{T_{ij}}{r}\right]_{t=t_e} d\mathbf{y} - \frac{1}{4\pi c_0^2}\frac{\partial}{\partial t}\iint_S \left[\frac{\rho\vartheta_i n_i}{r}\right]_{t=t_e} d\sigma$$
$$-\frac{x_j}{4\pi|\mathbf{x}|c_0^3}\frac{\partial}{\partial t}\iint_S \left[(P_{ij}+\rho\vartheta_i\vartheta_j)\frac{n_i}{r}\right]_{t=t_e} d\sigma \tag{44}$$

where $r = |x - y|$ and the retarded time t_e is

$$t_e = t - r/c_0 \simeq t - |\mathbf{x}|/c_0 \tag{45}$$

The first term in equation 44 is simply the turbulence noise as it would occur in absence of a foreign body (except for the fact that the control volume V excludes the body).

The second term is the result of movement of the body. For a rigid body at a fixed position we have

$$\vartheta_i n_i = \mathbf{v} \cdot \mathbf{n} = 0 \tag{46}$$

This term is important when the body is pulsating. For a compact body we have then a simple volume source term. This term can be used to describe the flow out of a pipe. Note that ρ is the fluid density just outside the control surface so that we consider the displacement of fluid around the body, rather than a mass injection.

The last integral in equation 44 is the momentum flux through the surface and the pressure and viscous forces. For a fixed rigid body $\rho\vartheta_i\vartheta_j = 0$ because $\vartheta = 0$ at a surface (no slip condition in viscous flow). In the case of a compact, fixed, and rigid body, we can neglect the emission time variation along the body, and we have $r \simeq |\mathbf{x}|$. The instantaneous force F_i of the fluid *on the body* (lift and drag) then:

$$F_i(t_e) \simeq \iint_S [P_{ij}]_{t=t_e} n_j \, d\sigma \tag{47}$$

Hence, for a fixed rigid compact body we have:

$$\rho'(\mathbf{x},t) = \frac{\partial x_i \partial x_j}{4\pi|\mathbf{x}|^3 c_0^4}\frac{\partial^2}{\partial t^2}\iiint_V T_{ij}(\mathbf{y}, t - |\mathbf{x}|/c_o)\, d\mathbf{y} - \frac{x_j}{4\pi|\mathbf{x}|^2 c_0^3}\frac{\partial^2}{\partial t^2} F_j(t - |\mathbf{x}|/c_o) \tag{48}$$

2.2 Aeolian Sounds

2.3 Cavity Sounds

2.4 State of the Art

3 Current Work

3.1 Wind Model

3.1.1 Model Description

This model presented here is a lightweight physical model of the sound created as air passes a solid cylinder. The equations presented in section 3.1.2 are taken from [8] which are reproductions of the relationships discovered by Strouhal. The variable wind speed model is taken from [9]. The underlying oscillation time is set to 100 seconds. The creation of gusts and squalls has not been altered.

A snapshot out the output signal is taken that converts the audio output to a signal. This is sampled from a metro object banging every 10 milliseconds. The output is then scaled by an underlying base wind speed value. The base wind speed value is set with an ordinary horizontal

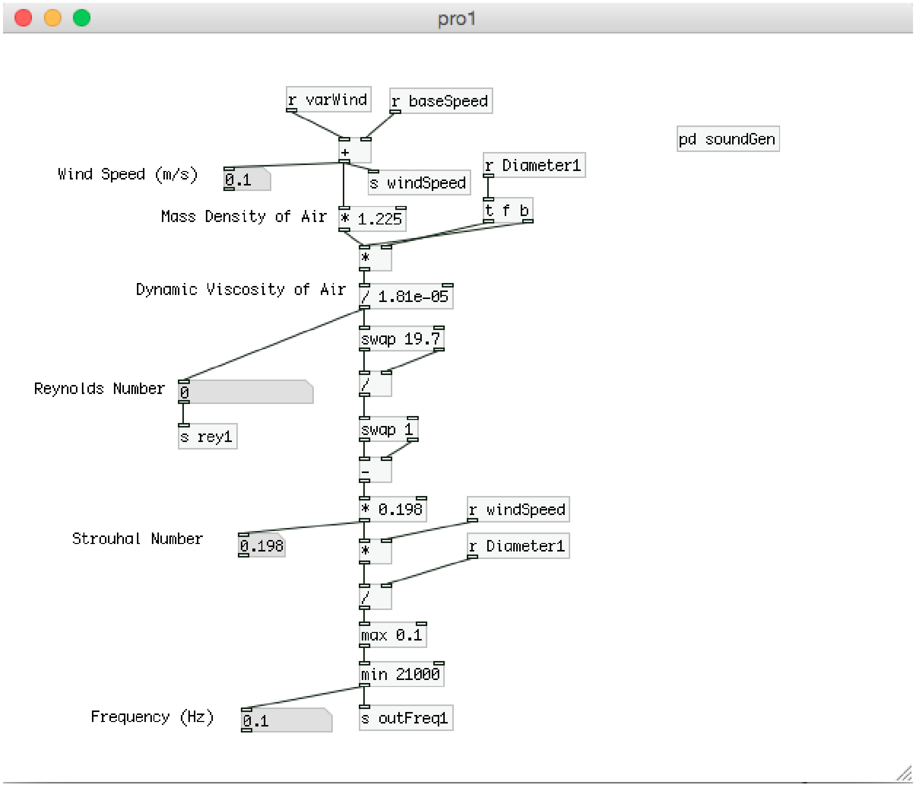

Figure 1: Pure Data patch calculating output frequency from physics

slider, ranging from 0 - 31 metres per second. With both the base wind speed and variable wind speed values are added together to give a range from approximately 0 - 39 metres per second. This ranges equates from no wind up to a hurricane.

The final wind speed value is used as an input to a patch that calculates the output frequency. A second input to the maths, (see subsection 3.1.2), is the diameter of the cylinder. The patch which calculates the frequency is shown in figure 1.

Once the frequency of the tone had been calculated the sound is generated. The patch showing the generation of this is shown in figure 2 and the q value of the bandpass filter is calculated in the patch shown in figure 3.

The final patch shows the patch that is presented to the user, figure 4. There are 4 cylinders that are available for the user to use. The user can change the diamater of each of the cylinders as well as adjust the underlying windspeed. The pan value of each cylinder has been set as can be seen in figure 2.

3.1.2 Maths behind the model

The equations used in this Pure Data model were taken from the original work by Vincenz Strouhal (1850-1922). The relationships used in the equations below were reproduced in [8] and shown below.

First we calculate the wind speed that is the force behind the sound source:

$$W_b[n] + W_v[n] = W[n] \tag{49}$$

where $W_b[n]$ is the current base wind speed, $W_v[n]$ is the current variable wind speed and $W[n]$ is the total wind speed. All values in metres per second.

The Reynolds Number gives an indication of the turbulence in the fluid flow. It is calculated as shown in equation 50:

$$R[n] = \frac{\rho_{air}\, d\, W[n]}{\mu_{air}} \tag{50}$$

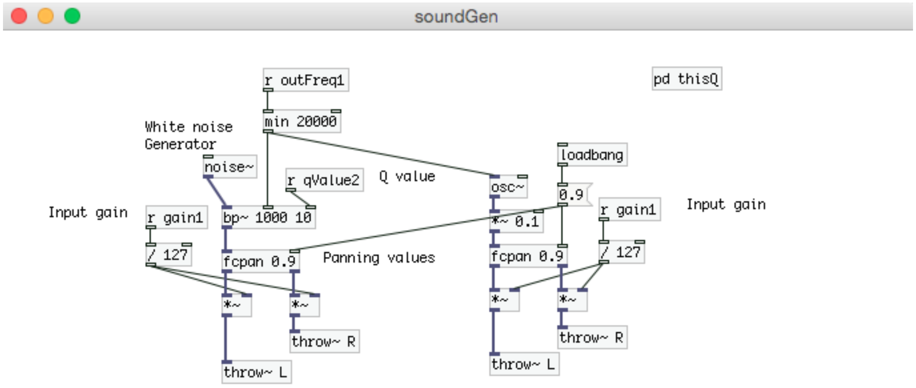

Figure 2: Pure Data patch calculating output sound

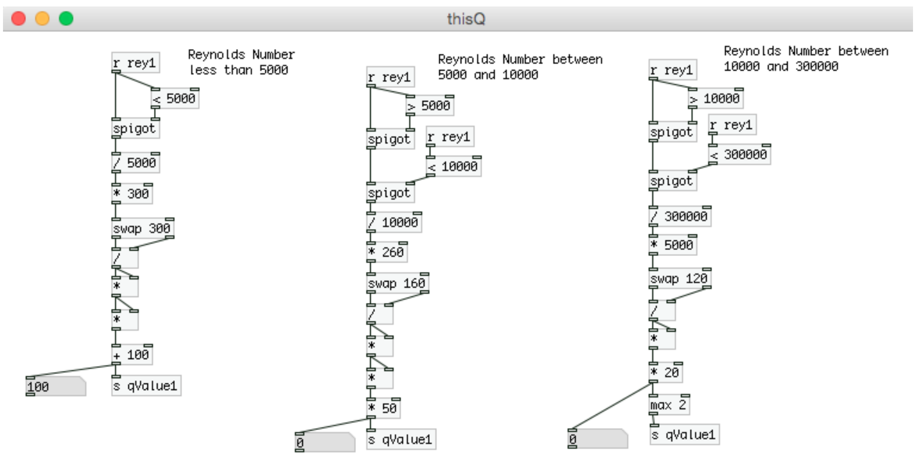

Figure 3: Pure Data patch calculating the Q value for the bandpass filter

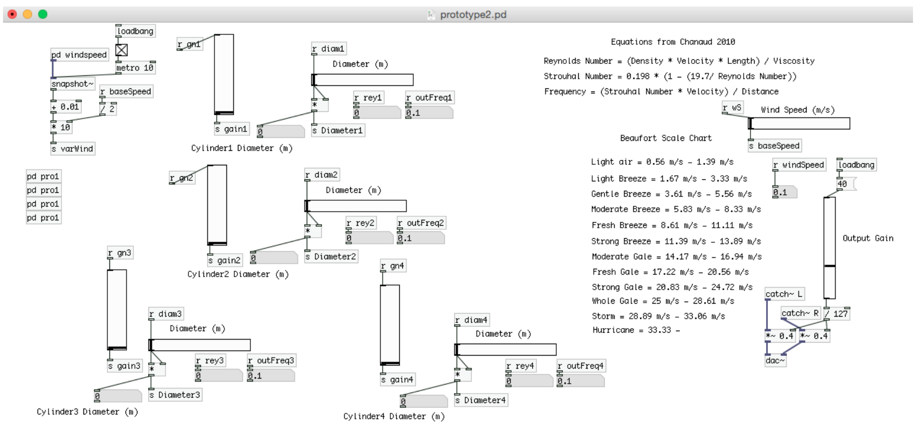

Figure 4: Pure Data patch showing parameters available for the user to interact with

where R[n] is the current dimensionless Reynolds Number, ρ_{air} is the Mass Density of air. This is given in [10] as 1.225 kilograms per square metre at sea level and 15°C. μ_{air} is the Dynamic Viscosity of Air, taken from [11] as $1.81 * 10^{-5}$ Newton - second per square metre. d is the diameter of the cylinder in metres.

Once the Reynolds Number is known it is possible to calculate the Strouhal Number:

$$S[n] = 0.198\,[1 - \frac{19.7}{R[n]}] \tag{51}$$

where S[n] is the current dimensionless Strouhal Number. With the Strouhal Number calculated it possible to calculate the frequency of the aeolian tone produced by the wind moving past a solid cylinder. This is given by:

$$f_t[n] = \frac{S[n]\,W[n]}{d} \tag{52}$$

where f_t[n] is the frequency of the aeolian tone. This value obtained for the frequency of the aeolian tone is then used as the input frequency of an oscillator, giving an output of:

$$Y_t[n] = 0.1\,cos[2\,\pi\,f_t[n]] \tag{53}$$

where Y_t[n] is the oscillator output. The value is multiplied by 0.1 to scale the tone the final output. As explained in [8], once the Reynolds Number increases about 5000 the pure tone changes to a spectrum with a central peak about the calculated frequency. In order to replicate this in Pure Data, a white noise source was mixed in with the oscillator. The noise was placed through a bandpass filter, with the centre frequency given by the calculated frequency and the Q value proportional to the Reynolds Number . The calculation for this is given below:

$$q = \begin{cases} [101, \infty], & Re[n] < 5000 \\ [7.5, 101], & 5000 < Re[n] < 10000 \\ [0.01, 9], & 10000 < Re[n] < 300000 \end{cases} \tag{54}$$

Thereafter the output of the bandpass filter is as follows:

$$B[n] = N[n]? \tag{55}$$

where N[n] is the output from a white noise generator and B[n] is the output of the bandpass filter with centre frequency determined by f_t[n] and q value set as in equation 54. The final output from the Pure Data patch to the digital to analogue converter is the sum of the pure tone and filtered noise outputs, as given in equation 56.

$$output[n] = B[n] + f_t[n] \tag{56}$$

3.1.3 Future Developments

One of the first points to highlight is that I am using one source per cylinder. In [12] similar sounds are created and they use a new sound source every three times the diameter. For example if the diameter of a cylinder is 5mm and 300 mm long, the number of sources required would be 20. This is worth further investigation and the differences of sound quality verses computational efficiency measured.

The main calculated parameter that is missing from the physical model is the calculation of the sound intensity from each source. This is calculated per source in [12] and it will be of benefit to examine this and implement this in Pure Data.

3.2 Sword Model

4 Future Work

5 Agenda

5.1 Year 2

5.2 Year 3

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