

The Melody Triangle: Exploring Pattern and Predictability in Music

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Abstract

The Melody Triangle is an interface for the discovery of melodic materials, where the input – positions within a triangle – directly map to information theoretic properties of the output. A model of human expectation and surprise in the perception of music, *information dynamics*, is used to ‘map out’ a musical generative system’s parameter space. This enables a user to explore the possibilities afforded by a generative algorithm, in this case Markov chains, not by directly selecting parameters, but by specifying the subjective *predictability* of the output sequence. We describe some of the relevant ideas from information dynamics and how the Melody Triangle is defined in terms of these. We describe its incarnation as a screen based performance tool and compositional aid for the generation of musical textures; the users control at the abstract level of randomness and predictability, and some pilot studies carried out with it. We also briefly outline a multi-user installation, where collaboration in a performative setting provides a playful yet informative way to explore expectation and surprise in music, and a forthcoming mobile phone version of the Melody Triangle.

Introduction

The use of generative stochastic processes in music composition has been widespread for decades—for instance Iannis Xenakis applied probabilistic mathematical models to the creation of musical materials (Xenakis 1992). However it can sometimes be difficult for a composer to find desirable parameters and navigate the possibilities of a generative algorithm intuitively.

The Melody Triangle is an interface for the discovery of melodic content where the parameter space of a stochastic generative musical process, the Markov chain, is ‘mapped out’ according to the *predictability* of the output. The Melody Triangle was developed in the context of *information dynamics* (Abdallah et al. 2012); an information theoretic approach to modelling human expectation and surprise in the perception of music. Users of the Melody Triangle do not select the parameters to generative processes directly, rather they provide input in the form of a position within a

triangle, and this maps to the information theoretic properties of an output melody. For instance one corner of the triangle returns completely random melodies, while an other area yields entirely predictable and periodic patterns, the entirety of the triangle covering a spectrum of predictability of the output melodies.

In this paper we outline the concepts and ideas behind information dynamics, and describe the information measures that lead to the development of the Melody Triangle. We describe its physical realisations; a multi-user interactive installation where visitors would use their bodies and gestures to generate musical materials, and a screen based interface. We outline some pilot studies carried out with the screen interface, as well as some qualitative feedback from music practitioners exploring its potential as a performance or composition tool. Finally we outline a forthcoming mobile phone version of the Melody Triangle.

Information Dynamics

The relationship between Shannon’s (1948) information theory and music and art in general has been the subject of some interest since the 1950s (Youngblood 1958; Coons and Kraehenbuehl 1958; Moles 1966; Meyer 1967b; Cohen 1962). The general thesis is that perceptible qualities and subjective states like uncertainty, surprise, complexity, tension, and interestingness are closely related to information-theoretic quantities like entropy, relative entropy, and mutual information.

Music is an inherently dynamic process. The idea that the musical experience is strongly shaped by the generation and playing out of strong and weak expectations was put forward by, amongst others, music theorists L. B. Meyer (1967a) and Narmour (1977).

An essential aspect of this is that music is experienced as a phenomenon that unfolds in time, rather than being apprehended as a static object presented in its entirety. Meyer argued that the experience depends on how we change and revise our conceptions *as events happen*, on how expectation and prediction interact with occurrence, and that, to a large degree, the way to understand the effect of music is to focus on this ‘kinetics’ of expectation and surprise.

Prediction and expectation are essentially probabilistic concepts and can be treated mathematically using probability theory. We suppose that when we listen to music, ex-

pectations are created on the basis of our familiarity with various styles of music and our ability to detect and learn statistical regularities in the music as they emerge. There is experimental evidence that human listeners are able to internalise statistical knowledge about musical structure, (Saf-ran et al. 1999), and also that statistical models can form an effective basis for computational analysis of music, (Con-klín and Witten 1995; Ponsford, Wiggins, and Mellish 1999; Pearce 2005).

Information dynamics considers several different kinds of predictability in musical patterns, how these might be quantified using the tools of information theory, and how they shape or affect the listening experience. Our working hypothesis is that listeners maintain a dynamically evolving probabilistic belief state that enables them to make predictions about how a piece of music will continue.

They do this using both the immediate context of the piece as well as using previous musical experience, such as a familiarity with musical styles and conventions. As the music unfolds, listeners continually revise this belief state, which includes predictive distributions over possible future events. These changes in probabilistic beliefs can be associated with quantities of information; these are the focus of information dynamics.

In this next section we briefly describe the information measures that we use to define the Melody Triangle, however a more complete overview of information dynamics and some of its applications can be found in (Abdallah and Plumbley 2009) and (Abdallah et al. 2012).

Sequential Information Measures

Consider a sequence of symbols from the viewpoint of an observer at a certain time, and split the sequence into a single symbol in the *present* (X_t), an infinite *past* (\overleftarrow{X}_t) and the infinite *future* (\overrightarrow{X}_t). The symbols arrive at a constant, uniform rate.

The *entropy rate* of a random process is a well-known, basic measure of its randomness or unpredictability. The entropy rate is the entropy, H , of the *present* given the *past*:

$$h_\mu = H(X_t | \overleftarrow{X}_t). \quad (1)$$

that is, it represents our average uncertainty about the present symbol *given* that we have observed everything before it. Processes with zero entropy rate can be predicted perfectly given enough of the preceding context.

The *multi-information rate* ρ_μ (Dubnov 2004) is the mutual information, I , between the ‘past’ and the ‘present’:

$$\rho_\mu = I(\overleftarrow{X}_t; X_t) = H(X_t) - H(X_t | \overleftarrow{X}_t). \quad (2)$$

Multi-information rate can be thought of as measures of *redundancy*, quantifying the extent to which the same information is to be found in all parts of the sequence. It is a measure of how much the predictability of the process depends on knowing the preceding context. It is the difference between the entropy of a single element of the sequence in isolation (imagine choosing a note from a musical score at random with your eyes closed and then trying to guess the

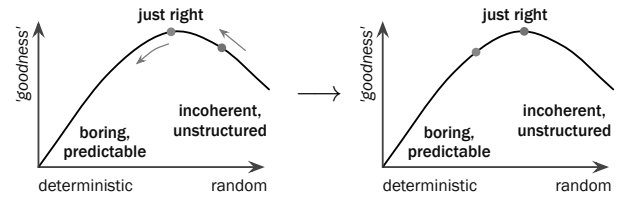


Figure 1: The Wundt curve relating randomness/complexity with perceived value. Repeated exposure sometimes results in a move to the left along the curve (Berlyne 1971).

note) and its entropy after taking into account the preceding context: If the previous symbols reduce our uncertainty about the present symbol a great deal, then the redundancy is high. For example, if we know that a sequence consists of a repeating cycle such as $\dots b, c, d, a, b, c, d, a \dots$, but we don’t know which was the first symbol, then the redundancy is high, as $H(X_t)$ is high (because we have no idea about the present symbol in isolation), but $H(X_t | \overleftarrow{X}_t)$ is zero, because knowing the previous symbol immediately tells us what the present symbol is.

The *predictive information rate* (PIR) (Abdallah and Plumbley 2009) brings in our uncertainty about the future. It is a measure of how much each symbol reduces our uncertainty about the future as it is observed, *given* that we have observed the past:

$$b_\mu = I(X_t; \overrightarrow{X}_t | \overleftarrow{X}_t) = H(\overrightarrow{X}_t | \overleftarrow{X}_t) - H(\overrightarrow{X}_t | X_t, \overleftarrow{X}_t). \quad (3)$$

It is a measure of the mutual information between the ‘present and the ‘future given the ‘past’. In other words, it is a measure of the *new* information in each symbol.

The behaviour of the predictive information rate make it interesting from a compositional point of view. The definition of the PIR is such that it is low both for extremely regular processes, such as constant or periodic sequences, *and* low for extremely random processes, where each symbol is chosen independently of the others, in a kind of ‘white noise’. In the former case, the pattern, once established, is completely predictable and therefore there is no *new* information in subsequent observations. In the latter case, the randomness and independence of all elements of the sequence means that, though potentially surprising, each observation carries no information about the ones to come.

Processes with high PIR maintain a certain kind of balance between predictability and unpredictability in such a way that the observer must continually pay attention to each new observation as it occurs in order to make the best possible predictions about the evolution of the sequence. This balance between predictability and unpredictability is reminiscent of the inverted ‘U’ shape of the Wundt curve (see Fig. 1), which summarises the observations of Wundt (1897) that stimuli are most pleasing at intermediate levels of novelty or disorder, where there is a balance between ‘order’ and ‘chaos’.

A similar shape is visible in the upper envelope of the plot in Fig. 3, which is a 3-D scatter plot of the information measures for several thousand first-order Markov

Though the interface is 2D, the third dimension (predictive information rate) is implicitly present, as transition matrices retrieved from along the centre line of the triangle will tend to have higher PIR.

As shown in as shown in Fig. 4, the corners correspond to three different extremes of predictability and unpredictability, which could be loosely characterised as ‘periodicity’, ‘noise’ and ‘repetition’. Melodies from the ‘noise’ corner (high h_μ , low ρ_μ and low b_μ) have no discernible pattern; those along the ‘periodicity’ to ‘repetition’ edge are all cyclic patterns that get shorter as we approach the ‘repetition’ corner, until each is just one repeating note. Those along the opposite edge consist of independent random notes from non-uniform distributions. Areas between the left and right edges will tend to have higher predictive information rate (b_μ), and we hypothesise that, under the appropriate conditions, these will be perceived as more ‘interesting’ or ‘melodic.’ These melodies have some level of unpredictability, but are not completely random. Or, conversely, are predictable, but not entirely so.

Given coordinates corresponding to a point in the triangle, we select from a pre-built library of random processes, choosing one whose entropy rate and redundancy match the desired values. The implementations discussed in this paper use first order Markov chains as the content generator, since it is easy to compute the theoretically exact values of entropy rate, redundancy and predictive information rate given the transition matrix of the Markov chain. However, in principle, any generative system could be used to create the library of sequences, given an appropriate probabilistic listener model supporting the estimation of entropy rate and redundancy.

The Markov chain based implementation generates streams of symbols in the abstract; the alphabet of symbols is then mapped to a set of distinct sounds, such as pitched notes in a scale. Further by layering these streams, intricate musical textures can be created. The Melody Triangle does not take into account the statistical experience of our exposure to tonal music. Even if a particular stream of symbols is periodic and predictable, in mapping to the chromatic scale there is a chance that the melody may conflict with culturally defined expectations. A mapping to the diatonic scale however is less likely to lead to such conflicts, and mappings to the pentatonic scale even less so. Indeed, the symbols can be mapped to a set of percussive sounds, and even non sonic outputs such as visible shapes, colours, or movements.

The information measures that define the Melody Triangle assume a constant rate of symbols, and thus the notes of each output melody proceeds at a uniform rate. Although the placing of events in time has a strong effect on expectations, surprise and satisfaction in music, the system does not, as yet, address this temporal dimension.

Interfaces

Interface 1: The Interactive Installation

The Melody Triangle was first implemented as a multi-user interactive installation. It has been exhibited at the Brighton Science Festival 2012, Digital Shoreditch as well as at The



Figure 5: The depth map as seen by the Kinect camera in the interactive installation version of the Melody Triangle. The bounding box outlines the blobs detected by OpenNI.

British Science Festival 2011. A Kinect¹ camera tracks individuals in a space, the range of its depth sensors naturally forming a triangle.

As visitors/users come into the range of the camera, they start generating a melody, the statistical properties of this melody determined by the mapping of physical space to the statistical space of the Melody Triangle. Thus by exploring the physical space, the participant changes the predictability of the generated melodic content. When multiple people are in the space they can cooperate to create interweaving melodies, forming intricate polyphonic textures.

This makes the interaction physically engaging and (as our experience with visitors both young and old has demonstrated) more playful.

Tracking and Control Tracking and control was done using the OpenNI libraries’ API² and high level middle-ware for tracking with Kinect. This provided reliable blob tracking of humanoid forms in 2D space. By triangulating this to the Kinect’s depth map it became possible to get reliable coordinate of visitors’ positions in the space.

By detecting the bounding box of the 2D blobs of individuals in the space, and then normalising these based on the distance of the depth map it became possible to work out if an individual had an arm stretched out or if they were crouching. With this it was possible to define a series of gestures for controlling the system without the use of any controllers (see table 1). Thus for instance by sticking out one’s left arm quickly, the melody doubles in tempo. By pulling one’s left arm in at the same time as sticking the right arm out the melody would shift onto the offbeat. Sending out both arms would change the instrument being ‘played’, and crouching would decrease the volume of the melody.

¹<http://www.xbox.com/en-GB/Kinect>

²<http://OpenNi.org/>

Table 1: Gestures and their resulting effect

left arm	right arm	meaning
out	static	double tempo
in	static	halve tempo
static	out	triple tempo
static	in	one-third tempo
out	in	shift to off-beat
out	out	change instrument
in	in	reset tempo

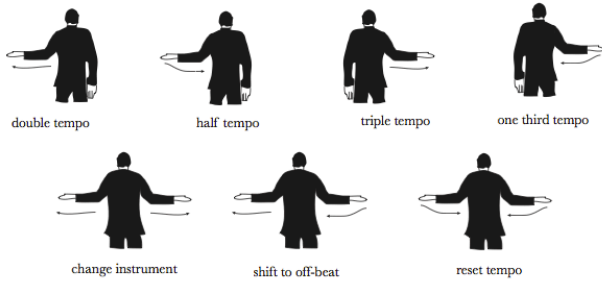


Figure 6: Gestures and their resulting effect

Observations Although visitors would need some initial instructions, they were then quickly able to collaboratively design musical textures. For example, one person would lay down a predictable repeating bass line by keeping themselves to the periodicity/repetition side of the room, while a companion can generate a freer melodic line by being nearer the 'noise' part of the space.

The collaborative nature of this installation is an area that merits attention. By not having one user be able to control the whole narrative, the participants would communicate verbally and direct each other in the goals of learning to use the system and finding interesting musical textures. This collaboration added an element of playfulness and enjoyment that was clearly apparent.

As an artefact this installation occupies an ambiguous role in terms of purpose; it is in a nebulous middle ground between instrument, art installation and technical demonstration. It is clear however, that as a vehicle for communicating ideas related to the expectation, pattern and predictability in music to the general public, it has proved very effective.

However we were interested in carrying out some studies under more controlled circumstances. Additionally we are interested in the Melody Triangle's potential as a compositional aid or music performance interface. To this end we developed a screen based user interface to the Melody Triangle.

Interface 2: The Screen Interface

In the screen based interface, a number of tokens, each representing a sonification stream or 'voice', can be dragged in and around the triangle. For each token, a sequence of symbols is sampled using the corresponding transition ma-

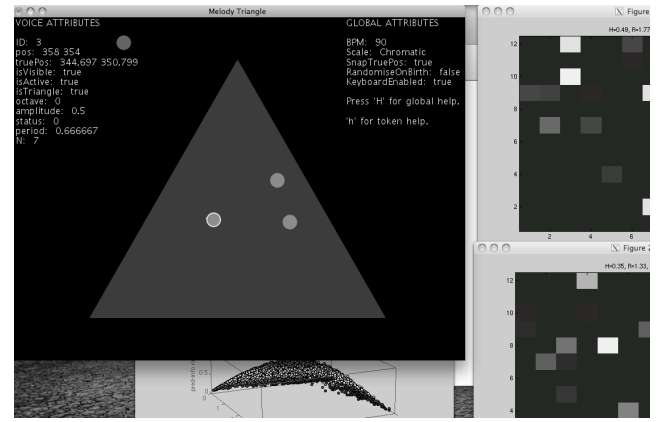


Figure 7: Screen shot of the Melody Triangle screen UI. On the right current transition matrixes being played are displayed. The tokens flash when ever a note from its melody is rendered.

trix, which are then mapped to notes of a scale or percussive sounds³. Keyboard commands give control over other musical parameters such as the pitch register, volume, scale, inter-onset interval and instrument for each voice. The system is capable of generating quite intricate musical textures when multiple tokens are in the triangle. The overlapping and interweaving of melodies of varying periodicity's and predictability is well suited for making content that could stylistically be characterised as 'minimalism'.

This interface is quite unlike other computer aided composition tools or programming environments, as here the composer exercises control at the abstract level of information-dynamic properties. A video of the interface in use can be viewed here - <http://bit.ly/My49IT>

User trials with the Melody Triangle

We carried out a pilot study with six participants who were asked to use a simplified form of the user interface (a single controllable token, and no rhythmic, registral or timbral controls) under two conditions: one where a single sequence was sonified under user control, and another where an additional sequence was sonified in a different register, as if generated by a fixed invisible token in one of four regions of the triangle. In addition, subjects were asked to press a key if they 'liked' what they were hearing. The subj

Our hypothesis is that users would linger longer in areas of the triangle that would produce more aesthetically desirable sequences, and these would tend to be the in the areas of the triangle that are of high predictive information rate, that is, areas along the middle and lower edge of the triangle.

We recorded subjects' behaviour as well as points which they marked with a key press. After the study the participants were surveyed with the Goldsmiths Musical Sophistication Index(Müllensiefen et al. 2011) to elicit their prior

³The sampled sequence could easily be mapped to other musical processes, possibly over different time scales, such as chords, dynamics and timbres. It would also be possible to map the symbols to visual or other outputs.

musical experience, which varied broadly. The sample size, however, was too small to draw any statistically significant correlations between the collected data and the index.

Results

Some results for four of the subjects are shown in Fig. 8. We have not been able to detect any systematic across-subject preference for any particular region of the triangle.

Comments collected from the subjects suggest that the information-dynamic characteristics of the patterns were readily apparent to most: several noticed the main organisation of the triangle, with repetitive notes at the top, cyclic patterns along one edge, and unpredictable notes towards the opposite corner. Some described their systematic exploration of the space. Two felt that the right side was ‘more controllable’ than the left (a consequence of their ability to return to a particular distinctive pattern and recognise it as one heard previously). Two reported that they became bored towards the end, but another felt there wasn’t enough time to ‘hear out’ the patterns properly. One subject did not ‘enjoy’ the patterns in the lower region, but another said the lower central regions were more ‘melodic’ and ‘interesting’.

Discussion

Our initial hypothesis, that subjects would linger longer in regions of the triangle that produced aesthetically preferable sequences, and that this would tend to be towards the centre line of the triangle for all subjects, was not confirmed. However the subjects did seem to exhibit distinct kinds of exploratory behaviour. It is possible that the design of the experiment encouraged an initial exploration of the space (sometimes very systematic, as for subject (c)) aimed at *understanding* how the system works, rather than finding musical patterns. It is also possible that the system encourages users to create musically interesting output by *moving the token*, rather than finding a particular spot in the triangle which produces a musically interesting sequence by itself.

We plan to continue the trials with a slightly less restricted user interface in order make the experience more enjoyable and thereby give subjects longer to use the interface; this may allow them to get beyond the initial exploratory phase and give a clearer picture of their aesthetic preferences. In addition, we plan to conduct a study under more restrictive conditions, where subjects will have no control over the patterns other than to signal (a) which of two alternatives they prefer in a forced choice paradigm, and (b) when they are bored of listening to a given sequence.

Qualitative Feedback

In parallel to the pilot study, we have collected qualitative feedback from potential users of the screen interface. Here four participants were interviewed, all practicing musicians that use computers in music production or in performance. This is with a view to establish what features would be desired for any eventual further development of the interface, for instance as a VST instrument for inclusion in a standard audio production environment.

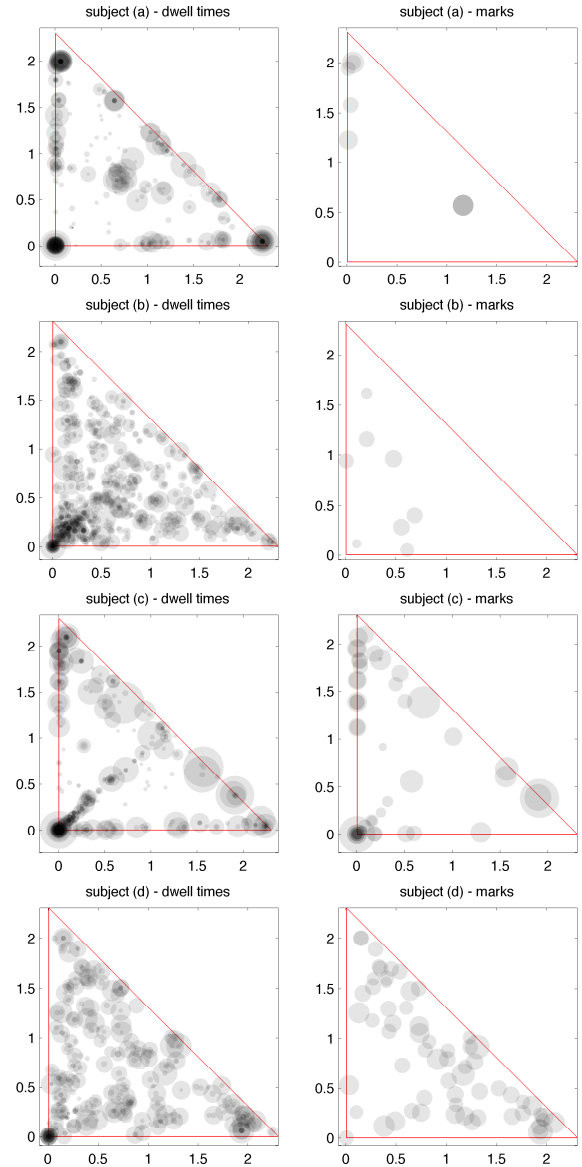


Figure 8: Dwell times and mark positions from user trials with the on-screen Melody Triangle interface, for four subjects. The left-hand column shows the positions in a 2D information space (entropy rate vs redundancy in bits) where each spent their time; the area of each circle is proportional to the time spent there. The right-hand column shows point which subjects ‘liked’; the area of the circles here is proportional to the duration spent at that point before the point was marked.

Unlike in the pilot study where participants would not know anything about the interface before hand and were asked to ‘explore’ with as little instructions in possible, here the potential users are first taught how to use the system. Then they are given some time to play and experiment, and in informal discussion feedback and criticism of the system is sought out. As part of a broader conversation, they were asked if they could identify the different areas of the triangle, what features of the system they liked and disliked, if they could see themselves using the system as part of their musical practice, and if so how.

Some points collected include -

- The subjects were very quick to get to grips with the properties of the different areas of the triangle, and found it quite intuitive.
- The more periodic/predictable half of the triangle was used considerably more by all participants.
- Some expressed interest in its potential as live performance interface for electronic music.
- All users desired more control over the mapping of symbols to notes, and some desired the ability to map the output of the triangle to other parameters such as to the control of filters and effect parameters.

Two of the users indicated that the Melody Triangle could integrate well into their musical practice, one was unsure and the other said it would not and expressed frustration at having little control over the musical style of the output. Some comments are provided here -

“If it was a kind of VST instrument, I would use it really a lot, definitely! Because there are not that many around that make this kind of stuff. I always love if something is generative or stochastic to generate things I would not come up with, but to generate a lot of them in a short amount of time and I’m the creative catalyst that just picks them.. and then have this kind of choices to edit probabilities, I really like that.”

“Here what is cool is that .. I can make multiple loops and they all have different characteristics and I don’t have to adjust like five numbers in different places, it’s in one thing, and that’s what I like most, it’s kind of like a macro [interface].”

“I would use it as an idea generator ..what i probably would do is I would run this, maybe I would select some random sounds and maybe I would try around and develop some motifs, and see ‘oh I like that!’ and would record that as midi and move on. ”

Stochastic process have often been used to generate musical materials. While such processes can drive the *generative* phase of the creative process, these comments suggest that information dynamics and the Melody Triangle can serve as a novel framework for a *selective* phase; helping composers discover generated materials that are of value. This alternation of generative and selective phases has been noted before (Boden 1990).

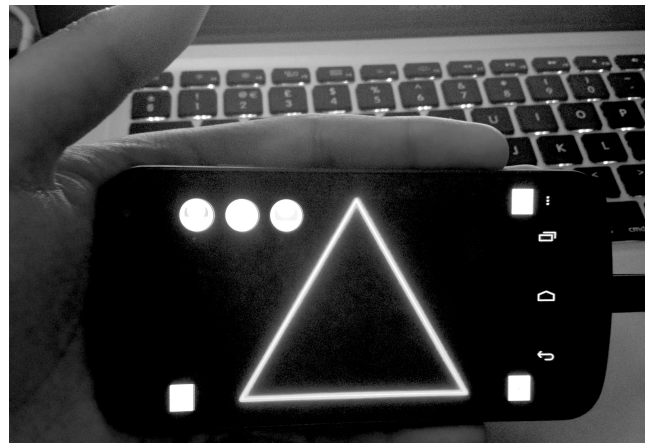


Figure 9: The Melody Triangle mobile phone app

The Mobile App

In order to further our study into musical preferences with a wider audience, the Melody Triangle is being implemented as an Android mobile phone application. The research motivation is to use the app as a means of collecting large quantities of crowd-sourced data, providing us with a larger data set than could be realistically achieved through individual studies.

The audio engine is developed in libpd⁴, a port of the open source Pure Data programming environment. The app will allow users to use the phone’s touch screen to drag tokens around the triangle and generate musical textures. Usage statistics will be collected on the phone and periodically uploaded to our servers for analysis.

Conclusion

We presented the Melody Triangle; an interface for the discovery of melodic content where the input – positions within a triangle – corresponds to the predictability of the output melodies. The Melody Triangle is contextualised in *information dynamics*; an information theoretic approach to modelling human expectation and surprise.

We outlined the relevant ideas behind information dynamics and described three key information theoretic measures; entropy rate, redundancy and a measure of *predictive information rate*, which describes the gain in information made by current observations about the future, but which are not already known from past observations. We described how the natural distribution of randomly generated Markov chains in terms of these measures lead us to design the Melody Triangle, and outlined its two physical incarnations.

The first is a multi-user installation where collaboration in a performative setting provides a playful yet informative way to explore expectation and surprise in music.

The second is a screen based interface where the Melody Triangle can be used as a musical performance interface or compositional aid for the generation of musical textures; the

⁴<http://libpd.cc/>

users control at the abstract level of randomness and predictability. We outlined some qualitative feedback gathered from users of the system. It indicates that the Melody Triangle could be useful as a performance tool or composition aid. We described a pilot study where the screen-based interface was used under experimental conditions to determine how the information dynamics measures might relate to musical preference. Although the results were inconclusive, we plan to continue this work under different experimental setups. Finally we outlined a forthcoming mobile phone version of the Melody Triangle that, when released, will collect data from its users with a view to help us identify any relationship between human musical preferences and the information-dynamic model of human expectation and surprise.

Acknowledgments

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